

Workshop on Symplectic Topology

Belgrade, August 2021

Thursday, August 19

10:30 - 11:30 **Filip Živanović : Exact Lagrangians from contracting C^* -actions**

In this talk, we will construct a family of closed exact Lagrangian submanifolds from contracting C^* -actions in complex symplectic manifolds. Moreover, we will compute Floer cohomologies of these Lagrangians. Finally, as an application, we will infer some lower bounds on the symplectic cohomology.

11:45 - 12:45 **Fabian Ziltener: A symplectic embedding of the cube with minimal sections and a question by Schlenk**

I prove that the open unit cube can be symplectically embedded into a longer polydisc in such a way that the area of each section satisfies a sharp bound and the complement of each section is path-connected. This answers a variant of a question by F. Schlenk.

14:45 - 15:45 **Maksim Stokić: Quantitative h -principle and isotropic flexibility in contact geometry**

The celebrated Eliashberg-Gromov rigidity theorem states that a diffeomorphism which is a C^0 -limit of symplectomorphisms is itself symplectic. Motivated by this, symplectic homeomorphisms are defined as C^0 -limits of symplectic diffeomorphisms. The contact version of Eliashberg-Gromov theorem holds as well, thus we can analogously define contact homeomorphisms as C^0 -limits of contact diffeomorphisms. Symplectic and contact homeomorphisms can either share a property with their smooth analogs (symplectic/contact rigidity), or they can behave in a flexible way. The powerful tool for proving flexibility statements in symplectic and contact geometry is the *quantitative h -principle*, which is an extension of the Gromov's h -principle adopted for applications in C^0 symplectic geometry. It was introduced first for symplectic 2-discs in 2016, and more recently for subcritical isotropic discs by Buhovsky and Opshtein. We prove a contact version of quantitative h -principle for subcritical isotropic discs. As an application, we show that the isotropic curves are C^0 -flexible in contact manifolds of dimension at least 5.

16:00 - 17:00 **Frol Zapolsky: Ideal-valued measures and symplectic rigidity**

In a joint work with A. Dickstein, Y. Ganor, and L. Polterovich, we introduce ideal-valued quasi-measures (IVQMs), a suitable generalization of Gromov's notion of ideal-valued measures on symplectic manifolds. These concepts allow us to put certain 'big fiber theorems' from topology and symplectic topology on equal footing. Moreover, I'll show how, using the quantum cohomology IVQM which we construct, we can prove new symplectic rigidity results. Finally, I'll discuss a relation between Entov-Polterovich's heavy sets and a new kind of rigid subsets, which we call SH-heavy, defined using the quantum cohomology IVQM. Our main tool is relative symplectic cohomology recently introduced by U. Varolgunes.

Friday, August 20

- 10:30 - 11:30 **Vukašin Stojisavljević : Oscillations of functions on surfaces via persistence barcodes**
 To a Morse function f on a closed manifold we associate a barcode by considering homologies of sublevel sets of f as a persistence module. Using the barcode, we define a quantity, $\Phi(f)$, which measures the total oscillation of f . More precisely, in dimension one, $\Phi(f)$ corresponds to the total variation of f , while in higher dimensions it relates to the integral of the Banach indicatrix. As the main result, we prove a Kronrod-Yomdin-type bound on $\Phi(f)$ in dimension two. This result can be applied to a certain question about uniform approximation of a function by linear combinations of Laplace-Beltrami eigenfunctions, possibly after a change of variables. The talk is based on a joint work with Iosif Polterovich and Leonid Polterovich.
- 11:45 - 12:45 **Gleb Smirnov : Symplectic boundary Dehn twist**
 Consider a symplectic manifold-with-boundary obtained from a closed symplectic manifold by removing a ball. A Dehn twist around the boundary contact sphere gives an element in the symplectic mapping class group rel boundary. I will show a few examples of how this element can be of infinite order. The talk, for the most part, will only assume elementary knowledge of algebraic topology.
- 14:45 - 15:45 **Dušan Joksimović : No symplectic-Lipschitz structures on $\mathbb{S}^{2n \geq 4}$**
 One of the central questions in C^0 -symplectic geometry is whether spheres (of dimension at least 4) admit symplectic topological atlas (i.e. atlas whose transition functions are symplectic homeomorphisms). In this talk, we will prove that the answer is “no” if we replace the word “topological” with “Lipschitz”. More precisely, we will prove that every closed symplectic-Lipschitz manifold has non-vanishing even degree cohomology groups with real coefficients. The proof is based on the fact that one can define analogs of differential forms and de Rham complex on Lipschitz manifolds which share similar properties as in the smooth setting.
- 16:00 - 17:00 **Mihajlo Cekić : Ruelle Zeta Function for nearly hyperbolic 3-manifolds**
 The Ruelle Zeta Function (RZF) is defined in analogy to the Riemann Zeta Function, where primes correspond to primitive closed orbits of an Anosov flow X . The RZF extends meromorphically to the whole complex plane and carries rich information about the flow. Using microlocal methods, Dyatlov-Zworski recently showed that the order of vanishing at zero $n(X)$ of the RZF equals to the minus Euler characteristic, if X is the geodesic vector field of a negatively curved surface. In this talk, I will explain an exciting novel result showing the instability of $n(X)$ close to hyperbolic 3-manifolds, starkly contrasting the case of surfaces. The proof is based on studying the pushforward of a certain pairing between resonant states (“eigenstates of X ”), regularisation arguments and wavefront set calculus. Joint work with Semyon Dyatlov, Benjamin Küster and Gabriel Paternain.